MATH 32 FALL 2012
MIDTERM 2 - SOLUTIONS

(1) (6 points) Find all values of x satisfying $2\log_5(x) + \log_{25}(x) = 5$.

Solution:

$$\begin{aligned}
2\log_5(x) + \frac{\log_5(x)}{\log_5(25)} &= 5 \\
2\log_5(x) + \frac{\log_5(x)}{2} &= 5 \\
\frac{5\log_5(x)}{2} &= 5 \\
\log_5(x) &= 2 \\
x &= 25
\end{aligned}$$

(2) A population of rabbits starts at 10 and doubles every 3 months.

(a) (6 points) Write down an expression for the rabbit population after t months have passed.

(b) (6 points) After how many months do you expect to have 200 rabbits? Write your answer as precisely as possible.

Solution:

(a) $P = 10 \cdot 2^{\frac{t}{3}}$.

(b) Solve $200 = 10 \cdot 2^{\frac{t}{3}}$. $2^{\frac{t}{3}} = 20$, $\frac{t}{3} = \log_2(20)$, so $t = 3\log_2(20)$.

(3) The equation $x^2 - 2x + 4y^2 + 24y + 33 = 0$ describes an ellipse.

(a) (6 points) Write this equation in the form $\frac{(x-h)^2}{a^2} + \frac{(y-v)^2}{b^2} = 1$.

(b) (3 points) What is the center of the ellipse?

(c) (3 points) What is the area of the ellipse?

Solution:

(a) Completing the square twice,

$$\begin{aligned}
(x-1)^2 - 1 + 4(y^2 + 6y) + 33 &= 0 \\
(x-1)^2 + 4(y+3)^2 - 36 + 32 &= 0 \\
\frac{(x-1)^2}{1} + 4\frac{(y+3)^2}{1} &= 4 \\
\frac{(x-1)^2}{2^2} + \frac{(y+3)^2}{1^2} &= 1
\end{aligned}$$

(b) $(1, -3)$

(c) $2 \cdot 1 \cdot \pi = 2\pi$.

(4) Consider the rational function

$$f(x) = \frac{(2x+4)(3x-1)(x+1)}{x^3+x}$$

You do *not* need to sketch a graph of f.

(a) (3 points) Does f have a horizontal asymptote? If so, what is it?
(b) (3 points) Does f have any vertical asymptotes? If so, what are they?
(c) (3 points) Does f have a y-intercept? If so, what is it?
(d) (3 points) Does f have any x-intercepts? If so, what are they?

Solution:

(a) Yes, the leading term of the numerator is $6x^3$ and the leading term of the denominator is x^3, so the horizontal asymptote is $y = 6$.
(b) Yes, the denominator factors as $x(x^2+1)$, x^2+1 has no zeros, and $x = 0$ is not a zero of the numerator, so there is one vertical asympotote, $x = 0$.
(c) No, since the function is not defined when $x = 0$.
(d) Yes, the zeros of the numerator are $x = -2, x = \frac{1}{3}$, and $x = -1$, and these are not zeros of the denominator, so the x-intercepts are $(-2, 0)$, $(\frac{1}{3}, 0)$, and $(-1, 0)$.

(5) (a) Find a point (x, y) so that $(4, -3)$ is the midpoint of the line segment connecting $(6, -\frac{9}{2})$ and (x, y).
(b) What is the length of this line segment?

Solution:

(a) We must have $\frac{x+6}{2} = 4$ and $\frac{y-\frac{9}{2}}{2} = -3$, so $x = 2$ and $y = -\frac{3}{2}$.
(b) The length of the segment connecting $(6, -\frac{9}{2})$ and $(2, -\frac{3}{2})$ is $\sqrt{(6-2)^2 + (-\frac{9}{2} - -\frac{3}{2})^2} = \sqrt{16+9} = \sqrt{25} = 5$.

(6) (6 points) Using the approximation formula $e^t \approx 1 + t$ when t is small, approximate the value of

$$\frac{e^{2.04}e^{3.002}}{e^5}$$

Solution: $e^{2.04+3.002-5} = e^{.042} \approx 1 + .042 = 1.042$.